TOWARD COMPUTATION OF VORTEX TYPE DUST COLLECTING APPARATUS

V. A. Uspenskii and V. I. Solov'ev

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A method for the computation of a vortex type dust collecting apparatus is proposed.

The construction of a vortex type dust collecting apparatus was proposed in [1] realizing great potentialities of dry mechanical methods of purification of gases. The model of the apparatus was investigated quite carefully [2-4] and it was established that in this apparatus dust particles larger than 5 μm are caught completely and the overall efficiency reaches 99.3-99.7%. The apparatus passed the industrial test and is in operation [5].

Unfortunately, as yet there are no techniques available for the computation of such an apparatus and the present work is an attempt in this direction.

The construction of the vortex type dust collecting apparatus is shown schematically in Fig. 1. The gas containing dust is led through a gas tube 1 to a vane swirler 2, curls around and arrives in the operating cavity of the apparatus. Under the action of the centrifugal forces the dust particles move to the peripheral layers, which is appreciably facilitated by the cowl 3. A gas (clean or contaminated without any preference) is fed into the ejector 4 tangentially to the cylindrical frame of the apparatus. The jet of the secondary gas helps to make the primary stream curl around; being directed at an angle to the generatrix of the frame it simultaneously blows away the peripheral layers of the gas saturated with dust into the bin 5, where the dust is collected. A washer 6 is placed at the exit from the apparatus.

We shall first determine the velocity fields in the operating cavity of the apparatus. We assume that the flow is laminar everywhere. Aiming at a qualitative description of the flow pattern in the apparatus we represent it as the sum of the following three flows: the rotational flow of the fluid in the cylinder above the fixed base, the flow of the fluid near the rotating disc, and the axisymmetric flow of the gas along the apparatus. We assume that in the present case the operating substance is the gas to be cleaned, a viscous incompressible fluid.

The first flow corresponds to velocity fields that are in steady state if the secondary gas is blown tangentially into a closed cylindrical container. The second flow will reflect the effect of the solid rotating

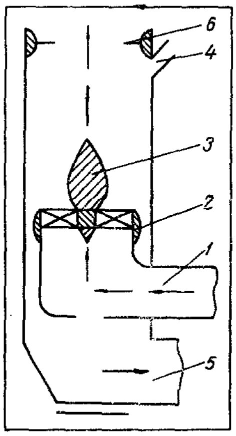

Fig. 1. Schematic diagram of vortex type dust collecting apparatus: 1) gas pipe; 2) vane swirler; 3) cowl; 4) ejector; 5) dust bin; 6) washer.

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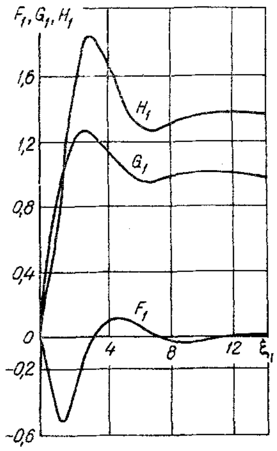

Fig. 2. Dependence of the functions F_1, G_1, and H_1 on ξ_1.

disc on the viscous gas, as the disc will impart rotational motion to the fluid. We should note the fact that in reality the rotational motion of the gas will result not only from the viscous friction, but also due to the momentum of the gas jets admixing with the flow; therefore, in order to estimate the effect of the vane swirler correctly it will be necessary to introduce an equivalent viscosity coefficient in the determination of velocity and temperature fields of the second flow. Finally, the third flow corresponds to the discharge of the operating gas through the apparatus as if there was no effect on the flow.

In the presence of heat transfer the first flow is described by the following equations:

$$u\frac{\partial u}{\partial r}+w\frac{\partial u}{\partial z}-\frac{v^2}{r}=-\frac{1}{\rho}\frac{\partial p}{\partial r}+\nu\left\{\frac{\partial^2 u}{\partial r^2}+\frac{\partial}{\partial r}\left(\frac{u}{r}\right)+\frac{\partial^2 u}{\partial z^2}\right\}, \tag{1}$$

$$u\frac{\partial v}{\partial r}+w\frac{\partial v}{\partial r}+\frac{uv}{r}=\nu\left\{\frac{\partial^2 u}{\partial r^2}+\frac{\partial}{\partial r}\left(\frac{v}{r}\right)+\frac{\partial^2 v}{\partial z^2}\right\}, \tag{2}$$

$$\frac{\partial u}{\partial r}+\frac{u}{r}+\frac{\partial w}{\partial z}=0, \tag{3}$$

$$u\frac{\partial w}{\partial r}+w\frac{\partial w}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial z}\pm\nu\left\{\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}+\frac{\partial^2 w}{\partial z^2}\right\}, \tag{4}$$

$$\rho g c_p\left(u\frac{\partial T}{\partial r}+w\frac{\partial T}{\partial z}\right)=\lambda\left\{\frac{\partial^2 T}{\partial r^2}+\frac{\partial}{\partial r}\left(\frac{T}{r}\right)+\frac{\partial^2 T}{\partial z^2}\right\}, \tag{5}$$

$$\frac{p}{\rho}=RT. \tag{6}$$

In a self-similar region of flow we shall seek the solution of this system of equations under the boundary conditions

$$u=0;\quad v=0;\quad w=0;\quad T=T_0\quad \text{for}\quad z=0;$$
$$u=0;\quad v=r\omega;\quad T=T_1;\quad p=p_0\quad \text{for}\quad z=\infty$$

in the form

$$u_1=r\omega_1F_1(\xi_1);\quad v_1=r\omega_1G(\xi_1);\quad w_1=\sqrt{\nu_1\omega_1}\,H_1(\xi_1);$$
$$T_1=rT_1(\xi_1);\quad p=r\omega_1\rho\sqrt{\nu_1\omega_1}\,P, \tag{7}$$

where $\xi_1=z\sqrt{\omega_1/\nu_1}$ is the new dimensionless coordinate.

After substituting (7) into the system of equations (1)-(5) we obtain the following system of equations in total derivatives for determining the five unknown parameters:

$$F_1^2+F_1'H_1-G_1^2-F''+1=0;$$
$$2F_1G_1+H_1G_1'-G_1''=0;$$
$$2F_1+H_1'=0; \tag{8}$$
$$P_1'+H_1H_1'-H''=0;$$
$$T_1''-H_1\,\mathrm{Pr}\,T_1'-F_1\,\mathrm{Pr}\,T_1=0$$

with the boundary conditions

$$F_1=0;\quad G_1=0;\quad H_1=0;\quad T_1=T_0;\quad P_1=P_0\quad \text{for}\quad \xi=0;$$
$$F_1=0;\quad G_1=1;\quad P_1=1;\quad T_1=T_0\quad \text{for}\quad \xi=\infty. \tag{9}$$

Pr is the Prandtl number.

The system of equations (8) is solved numerically and the values of the required functions are presented graphically in Fig. 2.

The second flow is described by the same equations as the first, only the boundary conditions are different:

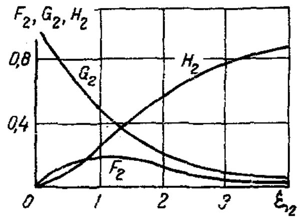

Fig. 3. Dependence of the functions F_2, G_2, and H_2 on ξ_2.

$$F_2 = 0;\quad G_2 = 1;\quad H_2 = 0;\quad T_2 = T_0;\ \Pr = P_0 \quad \text{for } \xi = 0;$$
$$F_2 = 0;\quad G_2 = 0;\quad T_2 = 1;\quad P_2 = 1 \quad \text{for } \xi = \infty. \tag{10}$$

The system of equations (1)-(5) with boundary conditions (10) is solved numerically and the results are given in Fig. 3. In this solution the dimensionless coordinate has the following form:

$$\xi_2 = z\sqrt{\frac{\omega_2}{\nu_2}}.$$

The third flow is described by the equation of discharge through the apparatus

$$Q_1 = \gamma_1 F W_{ax}. \tag{11}$$

Thus the velocity fields in the vortex type apparatus become known after a summation of the three investigated flows. It should be noted that these solutions are obtained for an infinite extent of the solid base over which the rotation of the viscous fluid occurs and for a rotating disc of infinite radius.

These solutions will be valid for the investigated conditions in the real apparatus, if the radius of the apparatus is large compared to the thickness of the layer entrained by the disc.

It has been experimentally established by the authors that the ratio of the thickness of the entrained layer to the radius of the apparatus is about 1 : 20.

For such a ratio the error of the solution does not exceed the accuracy attributed to engineering computations and the solution can be used in the computation of the apparatus.

Finally, we turn particular attention to the fact that the addition of the velocity fields, two of which are described by nonlinear differential equations, is not justified. This is perhaps the most vulnerable point in the solution of the formulated problem.

At present the satisfactory agreement between the computed and experimentally measured values of the velocities in the operating cavity of the apparatus permits the expectation that the nonlinear effects are small and effects that would appreciably deform the actual flow are not detected.

In analyzing the solutions obtained we should first note the presence of radial velocities which are especially significant in the presence of a strong vortex column. Radial flows to the center are highly undesirable, since they would carry the dust from the dust-laden peripheral layers to the center where the centrifugal forces are small. This phenomenon substantially impairs the efficiency of the dust catching apparatus. The first and the second solutions can be joined if radial flows of the gas toward the center are eliminated. Furthermore, the flow near the solid disc has an axial component of the velocity, which is directed opposite to the main flow; this helps to increase the stay of the gas in the apparatus. It is quite obvious that the second flow must be the prevailing flow, since it has a positive effect from all points of view. This is far from a trivial conclusion, which follows from the theory in the solution of the problem of redistribution of the expended energy between the primary and the secondary gas flows.

We write down the equation of motion of the dust particles with reduced diameter d and mass m. We shall assume that the forces acting on a dust particle in the plane normal to the axisymmetric flow in the apparatus are inertial, centrifugal forces, the forces due to pressure, and aerodynamic frictional forces. We also assume that the aerodynamic frictional forces can be represented by Stoke's formula, where as velocity we must use the algebraic sum of the radial velocities of the gas flow and the velocity of migration of the dust under the action of all external forces. The equation has the form

$$m\frac{dw_r}{dt} = m\frac{w_\tau^2}{r} - m\frac{1}{\rho}\frac{dP}{dr} - 3\pi d\mu\,(w_r - w_R), \tag{12}$$

$$w_\tau = \frac{1}{L}\int_0^L k_1\,(r\omega_1 G_1 + r\omega_2 G_2)\,d\xi, \tag{13}$$

$$w_R = \frac{1}{L}\int_0^L (r\omega_1 F_1 + r\omega_2 F_2)\,d\xi, \tag{14}$$

$$p=\frac{1}{L}\int_{0}^{L}\omega_1 r\rho\left[\sqrt{\nu_1\omega_1}P_1+\sqrt{\nu_2\omega_2}P_2\right]d\xi. \tag{15}$$

We substitute the values of the quantities occurring in Eq. (12) from (13), (14), and (15), and solve the resulting differential equation for zero boundary conditions:

$$w_r=\frac{B}{A}\left(1-e^{-At}\right), \tag{16}$$

where

$$A=3\pi d\frac{\mu}{m};\quad m=\frac{1}{6}\pi\rho_n d^3,$$

$$B=\frac{1}{L}\int_{0}^{L}\left[\omega_1(\sqrt{\nu_1\omega_1}P_1+\sqrt{\nu_2\omega_2}P_2)+3\pi d\frac{\mu}{m}(r\omega_1F_1+r\omega_2F_2)\right]d\xi+r\left[\frac{1}{L}\int_{0}^{L}(k_1\omega_1G_1+\omega_2G_2)\,d\xi\right]^2.$$

The time of stay of the dust in the apparatus, required for covering the distance from the axis to the wall, will be

$$t=\frac{D-D_{\text{co}}k_2}{2w_r}. \tag{17}$$

The washer at the exit increases the catching efficiency; as a result the path to the periphery is reduced and becomes smaller than the radius of the apparatus. At the same time it has been established experimentally that this path is larger than the path bounded by the radius of the washer.

For the present it is not possible to determine theoretically the actual effective distance of transfer of the dust to the wall of the apparatus and the improvement of the operation of the apparatus with the use of the washer must be regarded as an additional way of increasing the efficiency of the apparatus.

Substituting (17) into (16) we obtain

$$w_r=\frac{B}{A}\left[1-\exp\left(-A\frac{D-D_{\text{co}}k_2}{2w_2}\right)\right]. \tag{18}$$

In order that the time segment be positive, which is necessary for the displacement of the dust to the periphery, the following relation must hold:

$$t=\frac{L}{W_{\text{ax}}}=\frac{L}{\frac{4(Q_1+Q_2)}{\pi D^2\gamma}+\frac{1}{L}\int_{0}^{L}\left(\sqrt{\nu_1\omega_1}H_1+\sqrt{\nu_2\omega_2}H_2\right)d\xi}; \tag{19}$$

the left sides of Eqs. (17) and (19) must be identical. Hence

$$\frac{D-D_{\text{co}}k_2}{2w_r}=\frac{L}{\frac{4(Q_1+Q_2)}{\pi D^2\gamma}+\frac{1}{L}\int_{0}^{L}\left(\sqrt{\nu_1\omega_1}H_1+\sqrt{\nu_2\omega_2}H_2\right)d\xi}. \tag{20}$$

Formula (20) relates the main constructional quantities of the apparatus with the parameters characterizing the regimes of its operation.

The angular momentum of the rotating jet of the primary gas flow in the first flow is equal to the angular momentum introduced into the apparatus by the jet of the secondary gas:

$$\frac{Q_2}{g}W_{\max}D\cos\alpha=\frac{Q_1}{g}\frac{D^2}{4}\omega_1. \tag{21}$$

Hence

$$\omega_1=\frac{4Q_2W_{\max}\cos\alpha}{Q_1D}. \tag{22}$$

Formula (21) has been written without considering the interlayer friction of the gas of the main flow, but friction is taken into consideration by the expression determining $W_{\max}$ in accordance with (6):

$$W_{max} = W_e \left(\frac{0.343}{\frac{ax}{D_e} + 0.145} \right), \tag{23}$$

$$W_e = k_4 k_5 \sqrt{\frac{2g}{\gamma} \Delta P_e}, \tag{24}$$

$$x = \frac{L}{\sin\alpha}; \quad a = 0.017. \tag{25}$$

Substituting (23), (24), (25) into (22) we obtain

$$\omega_1 = \frac{4Q_2 k_3 k_4 k_5 \left(\frac{0.343}{\frac{aL}{D_e \sin\alpha} + 0.145} \right) \sqrt{\frac{2g}{\gamma} \Delta P_e}}{Q_1 D}. \tag{26}$$

The twist in the tube conducting the dust-laden gas can be determined from (7) with the aid of the expressions

$$\omega_p = \frac{2}{D_p} \cos\beta \sqrt{\frac{2g}{\gamma} H}\, k_6, \tag{27}$$

$$\omega_2 = \omega_p \frac{D_p^2}{D^2} = \frac{2D_p}{D^2} \cos\beta k_6 \sqrt{\frac{2g}{\gamma} H}. \tag{28}$$

Passing onto the estimate of the efficiency of the vortex type duct catching apparatus, we suppose that the degree of cleaning in the apparatus is directly proportional to the momentum of all external forces acting on the dust. Mathematically this can be expressed in the following way:

$$\eta_e = k \sum_{i=1}^{h} F_i t. \tag{29}$$

Since the momentum of the forces acting on the dust depends on the diameter of the dust particle and its mass, we can talk of the fractional efficiency only, and that is what (29) expresses.

Making use of formulas (12) and (17) and substituting (18) and (23)-(28) into them we obtain an expression in the most general form, making it possible to determine numerically the fractional (and later also the total) efficiency of the vortex type dust catching apparatus whose construction and operating regime are known:

$$\eta_e = k \frac{(D - D_{co} k_2)}{2 \frac{B}{A} (1 - e^{-At})} \left\{ \frac{1}{12} \pi d^3 \rho_n D \left[\frac{1}{L} \int_0^L (G_1 \omega_1 + G_2 \omega_2)\, d\xi \right]^2 \right.$$

$$\left. - \frac{1}{6} \pi d^3 \rho_n \frac{1}{L} \int_0^L \omega_1 \left(\sqrt{\nu_1 \omega_1} P_1 + \sqrt{\nu_2 \omega_2} P_2 \right) d\xi - 3\mu\pi d \frac{B}{A} (1 - e^{-At}) + 3\mu\pi d \frac{D}{2L} \int_0^L (F_1 \omega_1 + F_2 \omega_2)\, d\xi \right]. \tag{30}$$

Analyzing the composition of the physical quantities occurring in (30) we note that they can be divided into two groups: basic and specified. Among the basic parameters are: the discharge of the cleaned gas Q_1, the physicochemical properties of the gas and dust ν_1, ν_2, γ, ρ, ρ_n, μ, dispersion composition of the gas d_e. Besides, the a priori required efficiency of cleaning η is known to us from the conditions of satisfying sanitary norms. Experiments conducted by the authors have shown that some constructional characteristics of individual elements of the apparatus must satisfy the following relations:

$$D_p = (0.85 - 0.9)\, D; \quad D_{co} = (0.45 - 0.5)\, D;$$
$$D_w = 0.45D; \quad L = (3 - 4)\, D.$$

In order to reduce the computational work it is further necessary to specify the initial diameter of the apparatus D and its final value is refined in the process of the computation.

The remaining five parameters Q_2, H, ΔP_e, α, and β are determined from the system of five equations (20), (26), (28), and

$$\frac{\partial \eta}{\partial \Delta P_e} = 0, \tag{31}$$

$$\omega_1 F_1(\xi_1) = \omega_2 F_2(\xi_2). \tag{32}$$

The fulfillment of condition (31) enables one to choose a combination of parameters determining the feed of the secondary flow for minimum energy expenditure. Equation (32) guarantees the absence of radial velocities of the gas directed to the center of the apparatus. Finally, it is recommended that the parameters be chosen in the following ranges: $k = 1\text{–}1.12 \cdot 10^7$; $k_1 = 0.61\text{–}0.65$; $k_2 = 0.2\text{–}0.16$; $k_3 = 0.60\text{–}0.64$; $k_4 = 0.89\text{–}0.9$; $k_5 = 0.8\text{–}0.89$; $k_6 = 0.9\text{–}0.92$; $\nu_1 = 3.1 \cdot 10^6 \nu$; $\nu_2 = 4.2 \cdot 10^8 \nu$.

Quantities such as the discharge of the cleaned gas and the possible pressure drop in the vortex generator are related to the characteristic of the ventilator $Q_1 = f(H)$ and the computations must be carried out aimed at the existing means of draft blowing. On the one hand this restricts the set of typical dimensions of the apparatus, which can be constructed and introduced into the industry; on the other hand, the presence of the dependence $Q_1 = f(H)$ gives an additional condition substantially simplifying the computation of the apparatus.

NOTATION

u, v, and w	are the projections of the velocity on the coordinate axes;
p	is the pressure in the operating space of the apparatus;
r	is the radius of the apparatus;
ρ	is the mass density of the gas;
ν	is the kinematic viscosity of the gas;
T	is the absolute temperature;
λ	is the coefficient of thermal conductivity;
g	is the acceleration due to gravity;
c_p	is the specific heat;
H	is the pressure drop in the vortex generator;
G	is the discharge of the gas in terms of weight;
d	is the diameter of dust particle;
m	is the mass of dust particle;
μ	is the dynamic viscosity of the gas;
ω	is the angular velocity;
F	is the area;
w_r	is the radial component of the velocity of gas;
w_τ	is the tangential component of the velocity of gas;
w_R	is the radial component of the velocity of dust;
t	is the time;
D	is the diameter of the apparatus;
D_{co}	is the diameter of the cowl;
W_{ax}	is the axial velocity in the apparatus;
W_{max}	is the maximum velocity of gas flow at the ejector;
D_e	is the diameter of the ejector;
D_w	is the diameter of the washer;
α	is the angle of inclination of the ejector;
β	is the angle of the vanes of the vortex generator;
ΔP_e	is the pressure drop in the ejector;
D_p	is the diameter of the pipe;
L	is the length of the apparatus;
k	is the coefficient of proportionality between the efficiency and the momentum of the forces;
k_1	is the coefficient of peripheral velocities;
k_2	is the coefficient of recovery of dust due to the cowl;
k_3	is the coefficient of friction of the jet against the wall;
k_4	is the loss coefficient in outflow of gas from the ejector;
k_5	is the nonuniformity factor of the velocity field beyond the ejector;
k_6	is the friction coefficient in the vane swirler.

LITERATURE CITED

1. Patent FRG 1092281 kl. 50e 3/01.
2. K. Schmidt, Staub, 23, No. 11 (1963).

3. M. Klein, Staub, 23, No. 11 (1963).
4. W. Nikel, Staub, 23, No. 11 (1963).
5. Bulletin Ferrous Metallurgy, No. 8 (1966).
6. L. G. Loitsyanskii, Mechanics of Liquid and Gas [in Russian], Fizmatgiz (1956).
7. B. S. Stechkin et al., Theory of Vane Machines [in Russian], Oborongiz (1956).